

8 was used to calculate the boundary-layer characteristics up to the first slot, assuming a fully developed turbulent boundary layer from the nose; this boundary-layer velocity profile was then combined with estimated slot exit velocity profiles having a shape similar to those measured in Ref. 4. The resultant velocity profile was input to the slot injection code of Ref. 6. The slot to freestream total temperature ratio was assumed constant and equal to 0.99.

The numerical finite-difference method of Ref. 6 was modified to include multiple slot injection. The effect of one, three, five and ten slots on the fuselage skin friction drag was investigated. Figure 1 shows schematically the velocity profile development downstream of a single tangential slot. In all cases the first slot was located at  $x=0.09$  with the spacing between the slots being 0.09. The effect of slot height ( $S$ ) was studied for the ten-slot case only; slot heights of 3.65, 7.46, and 15.08 cm were investigated with the boundary-layer thickness ( $\delta$ ) at the first slot equal to 3.61 cm. The slot lip thickness was held constant at 0.16 cm. The input slot velocity profiles for the different slot heights were scaled by the heights of the slot; therefore the actual slot mass flow was proportional to  $S$ .

### Results and Discussion

The local skin friction coefficients ( $C_f$ ) obtained downstream of one, three, five, and ten slots (for  $S=7.46$  cm and the ratio of maximum slot velocity to stream velocity ( $U/U_\infty$ ) equal to 0.34) are compared with the local skin friction coefficient on the fuselage without slots ( $C_{f,0}$ ) in Fig. 2. The  $C_f$  reduction with only one slot is significant. The beneficial effect of the slot injection is most pronounced immediately downstream of the slot exit and diminishes with increasing surface distance ( $\ell$ ) downstream from the slot ( $\ell_T$  is the surface distance over the entire fuselage); this occurs because in the near slot region the wall friction is influenced only by the slot flow while further downstream mixing between the high momentum boundary-layer flow and the relatively low momentum slot flow increases the wall shear. The asymptotic level of skin friction (with injection) approaches a value which is less than that for no slots at moderate downstream distances. This final level depends among other things on the number of injection slots ( $N$ ).

The benefit of slot injection for  $C_f$  reduction is more meaningful in terms of total or average skin friction. The average skin friction coefficient is defined as

$$C_F = \frac{1.0}{\Delta \ell} \int_{\ell_s}^{\ell_T} C_f d\ell$$

where  $\ell_s$  is the surface distance to the first slot and  $\Delta \ell$  is the difference between  $\ell_s$  and  $\ell_T$ . The ratio of the reduction in average skin friction ( $C_{F,0} - C_F$ ) to the average skin friction with no injection ( $C_{f,0}$ ) is here defined as the skin friction reduction effectiveness ( $\eta$ ).

$$\eta = (C_{F,0} - C_F) / C_{F,0} = 1.0 - \left( \int_{\ell_s}^{\ell_T} C_f d\ell / \int_{\ell_s}^{\ell_T} C_{f,0} d\ell \right)$$

Figure 3 shows  $\eta$  as a function of  $N$  for  $S=7.46$  cm. The values of  $\eta$  for 10 slots with  $S$  equal 3.65 cm and 15.08 cm are also shown in Fig. 3 to illustrate the improvement in  $\eta$  with increased  $S$  and corresponding mass flow. It is obvious from Fig. 3 that large reductions ( $\approx 50\%$ ) in viscous drag are available through the use of slot injection systems.

The results shown in Figs. 2 and 3 suggest that the skin friction reduction ( $\eta$ ) is improved by increasing the number of injection slots but at a diminishing rate (for constant slot spacing, Fig. 3). One probable reason for this is that slot location is very important; for the present study, the most forward slot is the most effective and the most rearward slot is the least effective. Two advantages of a forward slot location in the present study are: 1) local  $C_{f,0}$  is high; and 2)  $\delta$  is small

[ $C_f$  reduction is improved at low values of  $\delta/S$  (Refs. 9 and 10)]. A forward location for slot injection offers the obvious additional advantage that drag reduction occurs over a large area of the aircraft. This effect of slot location is illustrated by the following comparison from Fig. 3; consider the case of 10 slots with  $S=3.65$  cm compared with the case of 5 slots with  $S=7.46$  cm (the first slot in each case is located at the same position). Although the total mass flow from the five 7.46 cm slots would be approximately the same as that for the ten 3.65 cm slots, the skin friction reduction is 27% greater for the 5 slot configuration than for the 10 slots (see comparison in Fig. 3).

In summary, the results of the present numerical study indicate that substantial skin friction reductions can be obtained with multiple slot injection on a fuselage shape representative of current long-haul subsonic transports. Although the results are very encouraging, they have not been experimentally validated. Systems penalties for collecting, ducting, and injecting the slot air have not been included and will significantly reduce the total available drag reduction. However, optimization of the slot geometry and turbulence control techniques may provide even greater skin friction reductions to help compensate for system losses. In addition, laminar flow control on the wings may provide a viable, low loss, air source for slot injection on the fuselage.

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## Supersonic Inlet Contour Interpolation

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### Nomenclature

$M$  = design Mach number (freestream)  
 $p$  = surface static pressure

Received June 30, 1975.

Index category: Airbreathing Propulsion, Subsonic and Supersonic.

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- $p_\infty$  = freestream static pressure  
 $p_{t\infty}$  = freestream total pressure  
 $R_L$  = cowl lip radius  
 $r$  = radius  
 $r' = r/R_L$   
 $X_L$  = cowl lip station at  $M$   
 $x$  = axial station measured from spike tip  
 $x' = x/R_L$   
 $\delta$  = flow angle relative to the inlet centerline, deg

#### Subscripts

- $th$  = throat  
 1 = first known inlet  
 2 = second known inlet  
 3 = interpolated inlet

### Introduction

THE method of designing original supersonic inlet contours generally has been an iterative process—that is, contours are input to a computer program that describes the supersonic flowfield. The output from the program is then examined to see if the desired surface compression rates and throat flow properties exist and are compatible with high performance. If not, the contours are modified, and new flowfield properties are computed. The process is repeated until the desired properties are achieved. The process is often tedious and time consuming. Hence, a new method of designing supersonic inlet contours has been investigated that reduces the tediousness and time required. Presently, the new method, which has been investigated for only axisymmetric, mixed-compression, supersonic inlets, merely requires interpolation of the contours of two known inlets designed for different Mach numbers to determine the contours for a third inlet at an intermediate design Mach number.

### Method of Interpolation

Two similar,<sup>‡</sup> known inlets are first selected, each designed for different Mach numbers. A percentage scale of length is generated for the centerbody and cowl (see Fig. 1) of each inlet so that 0% is at the spike tip for the centerbody and at the lip for the cowl, and 100% length is at the throat station (centerbody and cowl). At equal percentages of length for both the centerbody and cowl, axial stations are linearly interpolated with design Mach number for the new (or third) inlet, as indicated

$$x'_3 = (M_3 - M_1) [(x'_2 - x'_1) / (M_2 - M_1)] \times x'_1$$

Next, at equal percentages of length, third radii for both the centerbody and cowl are linearly interpolated

$$r'_3 = (M_3 - M_1) [(r'_2 - r'_1) / (M_2 - M_1)] + r'_1$$

Even though linear interpolation of the radii may not be rigorous, good results have been obtained (as for the following example) for relatively small differences in design Mach number ( $<0.5$ ) for the known inlets.

For larger differences in design Mach number ( $\sim 1.0$ ), it was found that linear interpolation was less accurate. That is, the throat area (contraction ratio) of the third inlet resulting from linear interpolation of the radii was either too large or too small, and the desired throat Mach number was not achieved. To overcome this problem, the throat area of the third inlet was fixed to achieve the desired throat Mach number, and an alternate, but more complex, method of interpolating radii was developed, but will not be discussed here.

<sup>‡</sup>Similar inlets mean inlets that "look" alike. Interpolation between external and internal compression inlets, or one axisymmetric inlet with smooth contours and one with discontinuities, would not be considered similar.

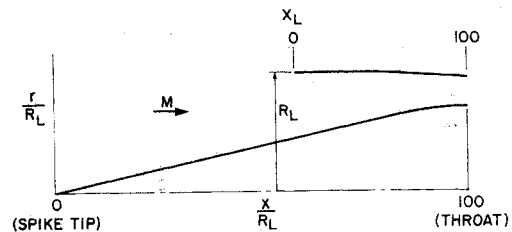


Fig. 1 Contour interpolation.

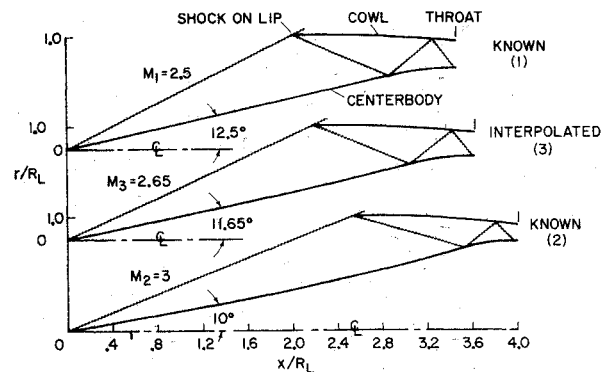


Fig. 2 Inlet contours.

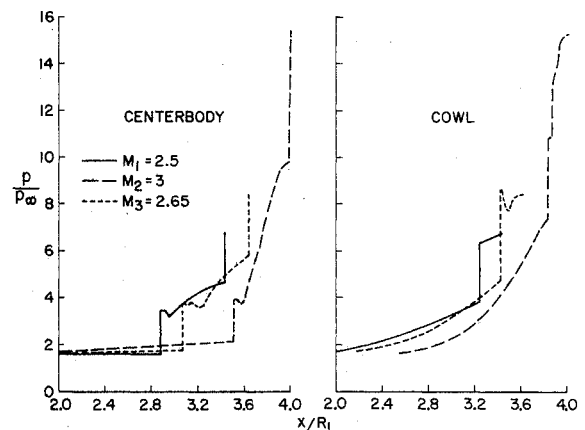


Fig. 3 Pressure distributions.

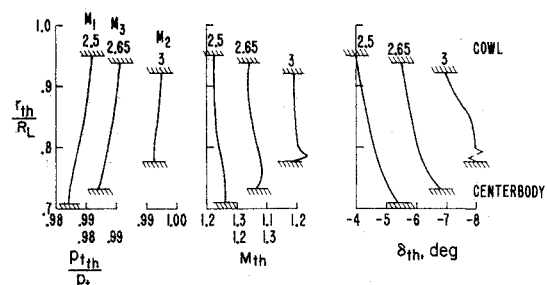


Fig. 4 Throat flowfield properties.

Note that, for the previous equations,  $M_3$  can be outside the range between  $M_1$  and  $M_2$ . Extrapolation for this possibility has not been investigated.

### Results

Several similar axisymmetric inlet contours have been interpolated from known inlets with design Mach numbers ranging from 2.16 to 4.0 and with design Mach numbers differing by as much as 1.0. An example of typical interpolation results is shown in Figs. 2-4. (The flowfields for all inlets

were calculated using the computer program of Ref. 1) Figure 2 shows the contours and internal shockwave structure for two similar known inlets<sup>2,3</sup> designed for Mach 2.5 and 3.0, respectively. Also shown are the resulting contours and shockwave structure for an interpolated inlet with a design Mach number of 2.65. Note that the bow shockwave from the 11.65° half-angle cone of the interpolated inlet falls on the cowl lip as it does for the known inlets. In addition, the interpolated inlet has a shockwave structure with the final shockwave impingement at the throat station on the centerbody similar to the known inlets.

With a similar shockwave structure, the interpolated inlet can be expected to have pressure distribution characteristics intermediate to the characteristics of the known inlets. This is shown in Fig. 3, where the pressure distributions on the cowl and centerbody are plotted for the three inlets shown in Fig. 2. Note that the distributions for the interpolated inlet are generally as smooth as for the known inlets and that the sudden pressure rises from the shockwave impingements are intermediate to the known inlets.

Finally, Fig. 4, the validity of the interpolation is shown by comparing the plots of the flowfield properties across the throat station of the interpolated inlet with the known inlets. The known inlets were iteratively designed with the goal of nearly isentropic pressure recovery, uniform flow angle, and a uniform Mach number of approximately 1.25 in the throat. Even though this goal was not ideally achieved for the known inlets, the interpolated inlet achieved similar shaped plots of these properties with an average pressure recovery of approximately 98.8%, an average Mach number of approximately 1.25, and a flow angle diverging from -5.5° on the cowl to -6.8° on the centerbody. Even the divergence in angle is favorable. This is because the flow must diverge in the throat to mate with a typical subsonic diffuser for further compression of the flow in an actual inlet system designed for a jet engine.

### Conclusions

It now seems possible to write a computer program so that a matrix of known inlet contours can be interpolated. The only inputs needed would be the selection of two known similar inlets in the matrix and the desired design and throat Mach numbers for the third inlet. With such a program, it seems reasonably certain that many useful supersonic inlet contours could be generated and investigated very simply and in a relatively short time. Even if the desired compression and throat properties were not achieved, the interpolation should provide a good start for further iterative adjustments to the contours and, at least, reduce the design time. Further, it may be possible to apply interpolation methods to other types of contours such as subsonic inlets and diffusers, nozzles, etc.

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## Errata

### Approximate Solution for Minimum Induced Drag of Wings with Given Structural Weight

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[J. Aircraft 12, 124-126 (1975)]

**E**QUATIONS (12) and (13) of the subject paper should read as follows:

1) Line 2 of Eq. (12) must read

$$\frac{C_2}{\pi} \eta^2 \mathcal{E}n\{ [1 - (1 - \eta^2)^{1/2}] / [1 + (1 - \eta^2)^{1/2}] \} -$$

i.e. square brackets and brackets shaped like this { } have to be included, and also the exponent  $\frac{1}{2}$  in the first term should be within round brackets.

2) In Eq. (13),  $\gamma Re$  must read  $\gamma_{Re}$ , i.e.  $Re$  has to be a subscript.

Received June 19, 1975.

Index category: Aircraft Aerodynamics (Including Component Aerodynamics).